Parallel Multilevel Methods for Implicit Solution of Shallow Water Equations with Nonsmooth Topography on Cubed-sphere

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Abstract. High resolution algorithms for the shallow water equations (SWE) on the sphere are very important for the modeling of the global climate. In this work, we introduce and study some highly scalable multilevel domain decomposition methods for the fully implicit (FI) solution of the nonlinear SWE discretized with a high order well-balanced finite volume method on the cubed-sphere. With the FI approach, the time step size is no longer limited by the stability condition, and with the multilevel preconditioners, good scalabilities are obtained on computers with a large number of processors.

1 Introduction

Numerical simulation of the shallow water equations (SWE) in spherical geometry lies at the root in the development of numerical algorithms for atmospheric circulation models. Spectral transform methods on latitude-longitude grid, providing high order accuracy, have been widely adopted for the spatial discretization of SWE in the last several decades. However, the high computational costs and communication overhead of the spectral transform methods become more and more severe for parallel supercomputers with distributed memories. The non-uniformity of the latitude-longitude grid results in load-imbalance in parallel simulations, especially when the grid is gradually refined. Besides, the singularities of latitude-longitude grid at the poles lead to substantial numerical difficulties in long-term calculations. Several efforts have been made on using numerical discretizations other than spectral method on composite grids instead of latitude-longitude grid. The composite grid is consisted of several pathes connected or overlapped together to cover the whole sphere, e.g., the icosahedron geodesic mesh ([1, 2]), the cubed-sphere mesh ([3]), the Yin-Yang mesh ([4]), among others. Although the cubed-sphere grid was proposed in early days ([3]), few work was done until it was revisited in 1990s ([5, 6]) and then drew increasing attentions thereafter ([7–18]).

For explicit or semi-implicit methods, due to stability limitations, the number of time steps increases as the grid is refined, which is unfavorable in terms of weak scalability. Ideal weak scalability only become possible when FI method is used, since the dependency between the time step size and the grid resolution is successfully removed. However, the price to pay using FI method is that a large sparse nonlinear algebraic system has to be solved at each time step. To solve the nonlinear systems efficiently, one often uses an inexact-Newton's method, within which Krylov-based iterative methods are used to solve Jacobian systems in the inner loop of each inexact-Newton's step. However, the increase of Krylov iterations, directly resulting in simulation time increase, neutralizes the benefits from the unconstrained FI time step size as the grid is refined ([19]). The only possible way to keep constant simulation time is to use a preconditioner which is not only inexpensive to apply but also capable to reduce the increased number of Krylov iterations to a reasonable level. Therefore, development of an effective and efficient preconditioner is crucial for a scalable FI solver, which is the goal of this study.

A one-level domain decomposition method was studied in [16] in preconditioning the linear systems arising from inexact-Newton's iterations to solve the nonlinear systems resulted from FI time integration of SWE on the cubed-sphere. The numerical results in [16] demonstrated that when a first-order finite volume scheme is used in spatial discretization, the parallel Schwarz preconditioner is robust both in terms of total linear iterations and computing time. However, for one-level Schwarz preconditioning, the number of linear iterations suffers as the time step size increases. The situation becomes even worse when higher order discretization is used, due to the fact that the resulting Jacobian system is even ill-conditioned. Besides, when topographic terms are included, the smoothness requirements for the nonlinear function in Newton's method are heavily violated and the convergence is unknown. In this paper we study an inexact Newton's method for the case with nonsmooth topography. The method is based on a semi-smooth technique to freeze multiple-valued points in the calculation of Jacobian matrices. Multi-level overlapping Schwarz methods based on low order Jacobian matrices are then used as preconditioners in solving the linear systems inside each inexact-Newton loop. We show by several benchmark cases that the FI solver offers good results in terms of both strong and weak scalabilities on machines with thousands of processors.

References

1. R. Sadourny, A. Arakawa, Y. Mintz, Integration of the nondivergent barotropic vorticity equation with an icosahedral-hexagonal grid for the sphere, Mon. Wea. Rev. 96 (1968) 351–356.
2. D. L. Williamson, Integration of the barotropic vorticity equation on a spherical geodesic grid, Tellus 20 (1968) 642–653.
3. R. Sadourny, Conservative finite-difference approximations of the primitive equations on quasi-uniform spherical grids, Mon. Wea. Rev. 100 (1972) 211–224.
4. A. Kageyama, T. Sato, Yin-Yang grid: an overset grid in spherical geometry, Geochem. Geophys. Geosyst. 5.
5. C. Ronchi, R. Iacono, P. Paolucci, The cubed sphere: A new method for the solution of partial differential equations in spherical geometry, J. Comput. Phys. 124 (1996) 93–114.

6. M. R. Rancic, J. Purser, F. Mesinger, A global-shallow water model using an expanded spherical cube: Gnomonic versus conformal coordinates, Quart. J. R. Met. Soc. 122 (1996) 959–982.
7. S. J. Thomas, R. D. Loft, Semi-implicit spectral element atmospheric model, J. Sci. Comput. 17 (2002) 229–350.
8. J. M. Dennis, A. Fournier, W. F. Spotz, A. St.-Cyr, M. A. Taylor, S. J. Thomas, H. Tufo, High resolution mesh convergence properties and parallel efficiency of a spectral element atmospheric dynamical core, Int. J. High Perf. Comput. Appl. 19 (2005) 225–235.
9. R. D. Nair, S. J. Thomas, R. D. Loft, A discontinuous Galerkin transport scheme on the cubed sphere, Mon. Wea. Rev. 133 (2005) 814–828.
10. R. D. Nair, S. J. Thomas, R. D. Loft, A discontinuous Galerkin global shallow water model, Mon. Wea. Rev. 133 (2005) 876–888.
11. J. M. Dennis, M. Levy, R. D. Nair, H. M. Tufo, T. Voran, Towards an efficient and scalable discontinuous Galerkin atmospheric model, in: Proceedings of the 19th IEEE International Parallel and Distributed Processing Symposium (IPDPS'05), 2005.
12. J. M. Dennis, R. D. Nair, H. M. Tufo, M. Levy, T. Voran, Development of a scalable global discontinuous Galerkin atmospheric model, Int. J. Comput. Sci. Eng. in press.
13. J. A. Rossmanith, A wave propagation method for hyperbolic systems on the sphere, J. Comput. Phys. 213 (2006) 629–658.
14. W. M. Putman, S.-J. Lin, Finite-volumn transport on various cubed-sphere grids, J. Comput. Phys. 227 (2007) 55–78.
15. C. Chen, F. Xiao, Shallow water model on cubed-sphere by multi-moment finite volume method, J. Comput. Phys. 227 (2008) 5019–5044.
16. C. Yang, J. Cao, X.-C. Cai, A fully implicit domain decomposition algorithm for shallow water equations on the cubed-sphere, SIAM J. Sci. Comput. 32 (2010) 418–438.
17. C. Yang, X.-C. Cai, A parallel well-balanced finite volume method for shallow water equations with topography on the cubed-sphere, J. Comput. Appl. Math. to appear.
18. C. Yang, X.-C. Cai, Newton-Krylov-Schwarz method for a spherical shallow water model, in: Proceedings of the 19th Intl. Conf. on Domain Decomposition Methods, 2009, to appear.
19. K. J. Evans, D. W. Rouson, A. G. Salinger, M. A. Taylor, W. Weijer, J. B. White, Iii, A scalable and adaptable solution framework within components of the community climate system model, in: ICCS 2009: Proceedings of the 9th International Conference on Computational Science, Springer-Verlag, Berlin, Heidelberg, 2009, pp. 332–341.